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**Mathematics Applications  
YEAR 12**

**Investigation 3 – Finance**

**Semester 2 2017**

**Take Home Section**

**Time allowed:** One week

**Marks Available:** No marks are allocated toward this section.

**Materials required:** Writing implements, correction fluid/tape or eraser, ruler, Scientific or CAS calculator

**Instructions:**

1. Write your answers in the spaces provided in this Question/Answer Booklet.
2. **Show all your working clearly in preparation for the Validation Test**. Your working should always be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Remember in the Validation Test, incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.
3. CAS calculators will be permitted to be used during the validation test.
4. No notes will be allowed.

This investigation examines the concept of interest.

**Definition of terms**

Define each of the following terms, in relation to interest, to clarify your own understanding:

Principal \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Interest \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Reducible Interest \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Repayment \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Per annum \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Quarterly \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Monthly \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Owing \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Opening Balance \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Closing Balance \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Loan \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Compound \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Interest Free \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Final Payment \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Reducible Interest**

When repayments are made on the money owing, the interest can be of the reducible type. The interest rate stays the same, but the amount of interest paid will be reduced, because repayments are being made. A housing loan is an example of a reducible loan.

Example 1 – Interest per annum

A loan of $15 000 is needed for a new car. The interest rate is 12% per annum and added yearly. Repayments of $3000 are made each year. How much is owed after 2 years?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | Principal | Interest | Principal +  Interest | Repayment | Amount owing |
| 1st year | 15 000 | 15 000 x 0.12 =  1 800 | 15 000 + 1 800 =  16 800 | 3000 | 16 800 – 3000 = $13 800 |
| 2nd year | 13 800 | 13 800 x 0.12 =  1 653 | 13 800 + 1 653 =  15 456 | 3000 | 15 456 – 3000 = $12 456 |

**After 2 years $12 456 is owed**

Example 2 – Interest per quarter

A loan of $15 000 is needed for a new car. The interest rate is 3% per quarter and added quarterly. Repayments of $750 are made each year. How much is owed after 2 years?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time | Principal | Interest | Principal +  Interest | Repayment | Amount owing |
| 1st quarter | 15 000 | 15 000 x 0.03 =  450 | 15 000 + 450 =  15 450 | 750 | 15 450 – 750 = $14 700 |
| 2nd quarter | 14 700 | 14 700 x 0.03 =  441 | 14 700 + 441 =  15 141 | 750 | 15 141 – 750 = $14 391 |
| 3rd quarter | 14391 | 431.73 | 14 822.73 | 750 | 14 072.73 |
| 4th quarter | 14 072.73 | 422.18 | 14 494.91 | 750 | 13 744.91 |
| 5th quarter | 13 744.91 | 412.35 | 14 157.26 | 750 | 13 407.26 |
| 6th quarter | 13 407.26 | 402.22 | 13 809.48 | 750 | 13 059.48 |
| 7th quarter | 13 059.48 | 391.78 | 13 451.26 | 750 | 12 701.26 |
| 8th quarter | 12 701.26 | 381.04 | 13 082.30 | 750 | 12 332.30 |

**After 2 years $12 332.30 is owed**

Both of the examples used the same Principal and time period.

Describe the difference in the repayment and the interest rate.

Which example gives the best loan for the borrower? Why?

1. Emma borrowed $10 000 to buy her car. Interest is charged on the opening balance each month at a rate of 9% per annum. Emma repays $ 1000 each month (except for the final payment). The final payment cannot exceed the regular payments. The table below shows Emma’s account over the life of the loan.

What is the interest rate used in the table?

Why is this rate different to the annual rate?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Month | Opening Balance | Interest | Repayment | Closing Balance |
| 1 | $10 000.00 | $75 | $1 000.00 | $9 075.00 |
| 2 | $9 075.00 | $68.06 | $1 000.00 | $8 143.06 |
| 3 | $8 143.06 | $61.07 | $1 000.00 | $7 204.14 |
| 4 | $7 204.14 | $54.03 | $1 000.00 | $6 258.17 |
| 5 | $6 258.17 | $46.94 | $1 000.00 | $5 305.10 |
| 6 | $5 305.10 | $39.79 | $1 000.00 | $4 344.89 |
| 7 | $4 344.89 | $32.59 | $1 000.00 | $3 377.48 |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  |  |
| 13 |  |  |  |  |

1. Complete the table above to find how long Emma takes to repay the loan. State the amount of the final payment.
2. How much interest would Emma have paid for the loan? Show clearly how you obtained your answer.
3. Dan borrows $5 000 from the bank to purchase a car. The annual rate is advertised at 8% per annum for his personal loan.  
     
   Calculate the monthly interest rate.

Dan repays $ 200 at the end of each month. Interest is calculated monthly. The following table shows the progress of his loan on a monthly basis.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Month** | **Opening Balance** | **Interest for the month** | **Repayment** | **Amount owing at end of month** |
| 1 | $5 000.00 | $33.33 | 200 | $4833.33 |
| 2 | $4 833.33 | **a** | 200 | **b** |
| 3 |  | $31.10 | 200 | $4496.66 |
| 4 | $4496.66 | $29.98 | 200 | $4326.64 |
| 5 | $4326.64 | $28.84 | 200 | $4155.48 |
| 6 | $4155.48 | $27.70 | 200 | $3983.18 |
| 7 | $3983.18 | $26.55 | 200 | $3809.74 |
| 8 | $3809.74 | $25.40 | 200 | $3635.14 |
| 9 | $3635.14 | $24.23 | 200 | $3459.37 |
| 10 | $3459.37 | $23.06 | 200 | $3282.43 |
| 11 | $3282.43 | $21.88 | 200 | $3104.32 |
| 12 | $3104.32 | $20.70 | 200 | $2925.01 |
| 13 | $2925.01 | $19.50 | 200 | $2744.51 |
| 14 | $2744.51 | $18.30 | 200 | $2562.81 |
| 15 | $2562.81 | $17.09 | 200 | $2379.90 |
| 16 | $2379.90 | $15.87 | 200 | $2195.76 |
| 17 | $2195.76 | $14.64 | 200 | $2010.40 |
| 18 | $2010.40 | $13.40 | 200 | $1823.80 |
| 19 | $1823.80 | $12.16 | 200 | $1635.96 |
| 20 | $1635.96 | $10.91 | 200 | $1446.87 |
| 21 | $1446.87 | $9.65 | 200 | $1256.51 |
| 22 | $1256.51 | $8.38 | 200 | $1064.89 |
| 23 | $1064.89 | $7.10 | 200 | $871.99 |
| 24 | $871.99 | $5.81 | 200 | $677.80 |
| 25 | $677.80 | $4.52 | 200 | $482.32 |
| 26 | $482.32 | $3.22 | 200 | $285.54 |
| 27 | $285.54 | $1.90 | 200 |  |
| 28 | **c** | $0.58 | **d** | 0 |
|  |  |  |  |  |

Determine the value of

a:

b:

c:

d:

How much did Dan actually pay for the car? (including interest)

Calculate the total amount of interest that Dan paid.

If the bank compounded interest on the daily balance, rather than charged simple interest on the monthly balance, would the total amount of interest have been less, more, or the same as above? Explain.

Dan could have borrowed the $5000 from his family on an ‘interest free’ basis, provided he pays it back in 2 years.

Assuming Dan makes equal monthly payments, calculate the minimum he would need to pay each month.

**End of Take Home Section of the Investigation**